

A Study on Blind Source Separation using ICA Algorithm in Terms of Invertible System

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Abstract—It is widely believed that in order to obtain the desired output from a composite system, the basic properties of the system plays an important role. While working with technologies like neural networking, Machine learning, Artificial Intelligence, Speech technology the basic properties became very prominent. This is supported empirically that the difficulty of recovering the hidden information or retrieving the original source signals from an unknown mixed signals, is a difficult process. In this paper, we explain one of the techniques of Blind Source Separation using Independent Component Analysis (ICA) in term of inevitability of the system and also prove the point that in order to implement Blind Source Separation using ICA algorithm the system needs to be invertible. To support the concept of inevitability of an ICA algorithm an experimental work is performed, where a neural network is designed to separate the source signals from a two component mixed signal. This paper is the comprehensive study of source separation using ICA which allow us to understand the basic fundamentals of ICA algorithm. The application of BSS in the field of Speech technology is limitless; it is widely used in noise separation, dimension reduction, encryption of hidden data.

Introduction

From the last two decades, with the growth of modern technological invention of high and fast computing system, scientists are basically focused in the field of machine learning and Artificial Intelligence and are trying to develop algorithms which make machines eligible to respond exactly as the human brain does. These algorithms include supervised Un-supervised and reinforcement learning. In this paper, we focus on supervised learning of speech signals for a problem known as Blind Source Separation using Independent Component Analysis in terms of invariability of the system.

A system is the interconnection of subsystems. A system can also be viewed as a process in which input signals are transformed by the system to behave or respond in some particular way. An application like chemical processing, signal processing, Machine Learning and Artificial intelligence required precise and high accuracy output [1]. All these are

possible with the help of the control system. For example, a high fidelity system takes a recorded audio signal and generates a reproducing signal. If the system has tone controls, we can change the tonal quality of the reproduced signal that is nothing but a control system [1]. Generally, the control system is LTIs (Linear Time Invariant) systems, and if we add the property of invariability with the control system it can be used for the variety of applications ranging from encryption of the original message for secure or private communication, Blind Source Separation (BSS) and so on [1]. In this paper, a comprehensive study on BSS has been made in terms of system and its analogy is made between the basic ideas of invariability of a system with Blind Source Separation.

Invertible System

A system can be considered as an invertible system if it obeys one-to-one mapping or a system is said to be invertible if its distinct inputs can produce distinct outputs [1]. In another term, it can also be defined as a system if we can get back the input $X(t)$ or $X[n]$ by passing the output $Y(t)$ or $Y[n]$ through another system called an inverse system. An invertible system must thus contain an inverse system. Fig.1(a) and Fig.1(b) shown below represents One-to-One mapping and Many-to-One mapping systems.

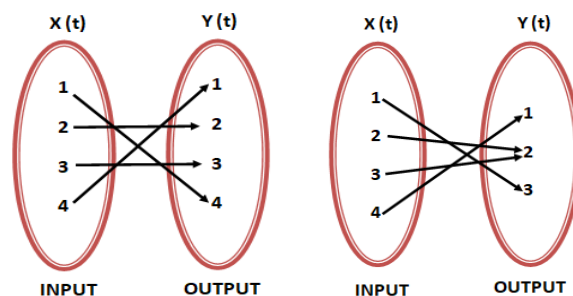


Fig.1(a): One-to-One Mapping

Fig.1 (b): One-to-Many Mapping

The basic block diagram of Invertible system is shown below:

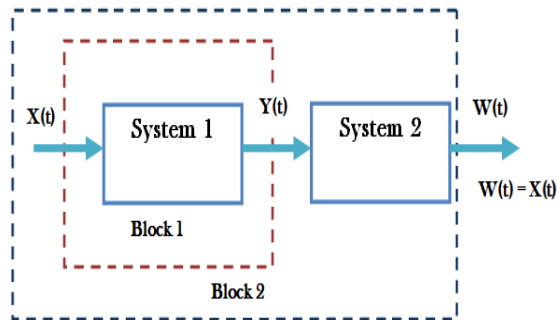


Fig. 2: Block diagram Representation of Invertible System

From the above block diagram, it can be seen that $W(t) = X(t)$, where $X(t)$ is the original input signal that is obtained at the final output of the composite system. Now **System 1** can be defined as an invertible system and **System 2** as its inverse. To elaborate the

$$\text{Let } Y(t) = 2X(t) \tag{1}$$

$$W(t) = \frac{1}{2} Y(t) \tag{2}$$

$$W(t) = X(t) \tag{3}$$

Networks that are invertible by construction offer a unique opportunity. This type of system can be used to train in both ways Supervised and Un-supervised resulted in higher accuracy and flexibility of the system [2]. To obtain the specification of an invertible system a class of neural networks has been introduced known as Invertible Neural Networks (INNs). These types of neural networks are different from Classical Neural Networks. Classical Neural Networks generally solves the ambiguities just inverting the problem directly, whereas INNs focuses on learning the forward process, using additional latent output. Because of the property of invertibility, a model of the corresponding inverse process can be learned completely [1-2].

Blind Source Separation

Blind Source separation refers to a problem where both the source and the mixing method are unknown, commonly known as the cocktail party problem. Consider a situation in which multiple source signals are interfering with one another. In this case, the mixed signals are not intelligible and it is of the prior interest to separate the individual signals. The problem of separating the source signals from the mixed signals is classically stated as cocktail party problem. The BSS is considered as a black box method. Since, the system will be unaware of the source as well as the medium of recording is also unknown. The time delay between microphones, echo amplitude difference, and undetermined mixture signal are some of the major problems encountered while implementing BSS [3].

If we consider a situation where a number of people are talking simultaneously in a room and we start recording through multiple microphones placed at different locations in the room. Then all the microphones will give us the recorded mixed signals which can be denoted by $x_1(t), x_2(t), \dots, x_N(t)$ where x_1, x_2, \dots, x_N are the amplitude and t is the continuous time index. Each of these recorded signals is a weighted sum of the speech signals emitted by the multiple speakers. Let us consider two microphones case-

$$X_1 = a_{11} S_1 + a_{12} S_2 \tag{4}$$

$$X_2 = a_{21} S_1 + a_{22} S_2 \tag{5}$$

where a_{11}, a_{12}, a_{21} and a_{22} are some parameters that depend on the distance of the microphones from the speakers. The basic idea of Blind Source Separation is to separate out the original signals through a system using machine learning techniques like Supervised and Un-supervised learning [3].

A typical block diagram of Blind Source Separation using ICA algorithm is shown below.

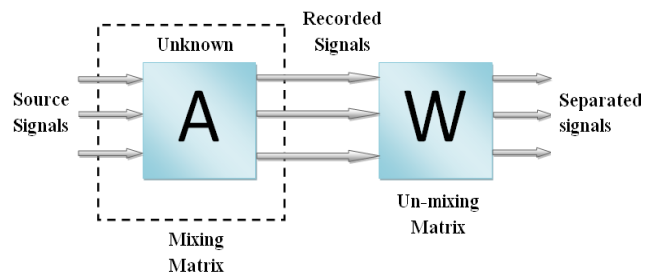


Fig. 3: Block diagram for Blind Source Separation.

Now if we compare the figures (2) and (3) there is some similarities in term of operation of the system, which is explained in **section (IV)**. Here the property of Invertibility of the system comes into picture. In order to implement ICA blind source separation the system should be invertible and there shall exist an inverse system for it.

In a Cocktail party, several recordings of the same signal are gathered during a reception. The problem of Cocktail party can be generalized as follows [4]:

$$X(k) = [x(k_1), x(k_2), x(k_3), \dots, x(k_T)] \in R^{T*1} \tag{6}$$

Where, k is the discrete component variable (time, distance). The aim is to find the inverse system in order to estimate the primary source signal given by:

$$S(k) = [s(k_1), s(k_2), s(k_3), \dots, s(k_T)] \in R^{T*1} \tag{7}$$

Most of the time we consider the following model where the sources are mixed linearly:

$$X = AS + V \tag{8}$$

$$X = [x_1, x_2, x_3, \dots, x_t] \in R^{m*T} \tag{9}$$

Where m is the number of available samples (mixtures) and T is the number of observations.

In equation(8), $A \in \mathbb{R}^{m \times n}$ denotes the unknown mixing and n denotes the number of sources; $V \in \mathbb{R}^{m \times T}$ is a mixing matrix representing the noise and 'S' contains the unknown source signal. BSS aims at finding the un-mixing matrix W such that [4]:

$$S = W * X \tag{10}$$

Basically, there are two main approaches (ICA and PCA) used for BSS but from the application point of view, it has been found that for speech Blind Source Separation ICA works far better than Principal Component Analysis. It is because in case of ICA, all the source components are considered mutually statistically independent [5].

There are some assumptions to be considered before implementing Blind Source Separation [5]:

1. The sources being considered are statistically independent.
2. The independent components have a non-Gaussian distribution.
3. The mixing matrix is invertible.

Among the three assumptions mentioned above the most appalling assumption is the invertibility of the matrix. Next, in **section (IV)** we tried to explain the invertible architecture of Blind Source Separation.

Invertible Architecture of Blind Source Separation using ICA

The most important block of a supervised Blind Source Separation using ICA is the reversible block which consists of a un-mixing matrix. The matrix defines the actual concept of Blind Source Separation. The retrieving of the original source signal from a mixed signal through an artificial neural network can be accomplished by reducing redundancy between the signals. This approach generally leads to an algorithm called Independent Component Analysis (ICA).

ICA is based on, one of the powerful assumptions that the different physical processes generate completely different signals [5]. In ICA source signals are separated, considering that the original source signals are distributed independently and there feature extraction data points are completely unique. ICA is often described as an extension to Principle Component Analysis (PCA) which generates a non orthogonal basis by un-correlating the signals for larger order moments and produces a non-orthogonal basis. In ICA algorithm the sources are generated linearly where additive noise can be present. [5].

Suppose we obtain a set of N observation signals

$X_i(t), i=1, 2, 3, \dots, N$ through sensors (microphone), that are mixtures of the sources, the main idea behind the mixing process is that the microphones can be randomly placed at

certain distance so that each sensor records a different mixture of the source signals. With the spatial separation assumption in mind, we can model the mixing process:

$$X = AY \tag{11}$$

The defined model can be considered as a system which produces a mixed signal from a set of observation signals. Where X and Y are the two vectors representing the observed signals and source signals respectively and A is an unknown matrix called the mixing matrix [5].

Here the main objective is to remove the original signals, S_i from the observed vector X_i . Which is accomplished by obtaining the un-mixing matrix W , the W matrix is designed such that $W = A^{-1}$. This enables to estimate the original source signals separately, S . The estimating matrix W can be considered an inverse system of the first system.

$$S = S' = W \times X \tag{12}$$

where S is original source signal and S' is the separated source signal.

Better the estimation of the un-mixing matrix better will be the approximation of the sources signals. The best condition of ICA to be executed is that the number of sources and sources are equal must be equal. Unfortunately, the ideal condition source separation might not be the situation always. There might be more two conditions where the numbers of the sensors are not equal to the number of sources; these are called Over-complete ICA and Under-complete ICA [5].

- 1) Over-complete ICA: It is an ICA source separation problem which occurs when the numbers of sources are more than number of recording sensors.
- 2) Under-complete ICA: It is an ICA source separation problem which occurs when the numbers of sources are less than number of recording sensors.

Concentrating on the case of Over-complete ICA where the number of sources exceeds the number of recording $x_1(t)$ and $x_2(t)$ from three independent sources $s_1(t), s_2(t)$ and $s_3(t)$. The coefficients depend on the distances between the sources and the sensors.

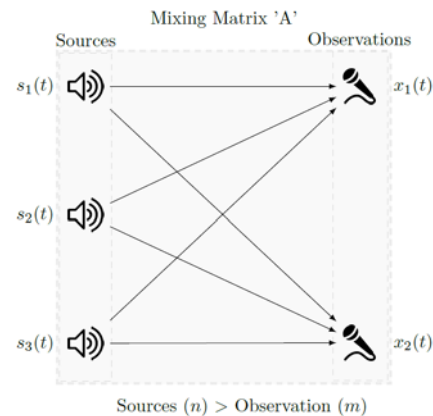


Fig. 4: Over-complete ICA

$$x_1(t) = a_{11}s_1(t) + a_{12}s_2(t) + a_{13}s_3(t) \tag{12}$$

$$x_2(t) = a_{21}s_1(t) + a_{22}s_2(t) + a_{23}s_3(t) \tag{13}$$

The a_{ij} are constant coefficient that gives the mixing weights. The mixing process of these vectors can be represented in the matrix form as shown below.

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix}$$

The un-mixing process and estimation of sources can be written as-

$$\begin{bmatrix} s'_1 \\ s'_2 \\ s'_3 \end{bmatrix} = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \\ w_{31} & w_{32} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Here, the matrix A of size 2×3 and un-mixing matrix W is of size 3×2 . Hence computation of sources in over-complete ICA requires some estimation processes [5].

Let us consider,

$$x_1(t) = a_{11}s_1(t) + a_{12}s_2(t)$$

$$x_2(t) = a_{21}s_1(t) + a_{22}s_2(t)$$

Here,

$$x_1(t) \neq x_2(t) \tag{14}$$

Since all the coefficients say a_{ij} ; $i,j=1, 2, 3 \dots N$ are randomly generated weights.

Suppose we have,

$x_1(t), x_2(t), x_3(t), \dots, x_N(t)$ numbers of observation signals.

$s_1(t), s_2(t), s_3(t), \dots, s_N(t)$, numbers of source signals then mathematically can be written as-

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = A_{3 \times 3} \begin{bmatrix} s_1(t) \\ s_2(t) \\ s_3(t) \end{bmatrix} \tag{15}$$

$$X_{3 \times 1} = A_{3 \times 3} S_{3 \times 1} \tag{16}$$

Where,

$$X = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}, S = \begin{bmatrix} s_1(t) \\ s_2(t) \\ s_3(t) \end{bmatrix},$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \dots & \vdots \\ \vdots & \vdots & \dots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

Equation (16) is the invertible system because of the following observations:

1. $A_{m \times n}$ is randomly generated mixing matrix, that is $s_1 \rightarrow x_1, s_2 \rightarrow x_2, \dots, s_N \rightarrow x_N$.

2. It also satisfies the one-to-one mapping.

Since the above equation can be claimed to be the invertible equation, so there must exist an inverse equation. The only parameter in the Equation (16) which defines the characteristics of the invertibility of the system is the mixing matrix A . And ICA algorithm performed BSS by designing an inverse system by estimating the un-mixing matrix.

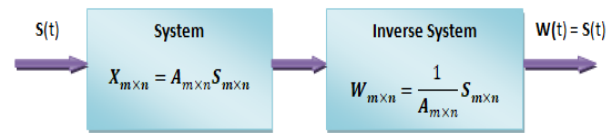


Figure 5: Diagram of final analogy of BSS with invertible system.

Similarly for the inverse system considering the ideal case where $(n=m)$,

$$M = S' = W_{m \times n} X_{m \times n} \tag{17}$$

$$W_{m \times n} = \frac{1}{A_{m \times n}} \tag{18}$$

$$S'_{m \times n} = W_{m \times n} X_{m \times n} \tag{19}$$

Equation (19) is the inverse Equation. Equating Equation (16) and (19) the original source signals can be retrieved back with some ambiguities like amplitude and order. Since the composite system of supervised Blind Source Separation consists of a system which has an inverse system of it. Hence, supervised Blind source Separation System using ICA algorithm can be called an invertible system.

Experimental Details

Essential requirement needed to perform Blind source separation (BSS) are given below:

- a. *Collection and preparation of data-set:* Raw speech signals (original source signal) have been recorder with microphones and then it was mixed through a mixer having a fundamental frequency of 8KHz mono. The original source signals are used for the training of the neural network while mixed data is used for testing. The raw data is then converted in Zip format.
- b. *Training:* During the training of the neural network the Zip data is labeled say (signal_1=0, signal_2=1, signal_3=2, ..., signal_n=N) and then feed to the neural network through ICA algorithm.
- c. *Testing:* Finally for testing, the mixed raw data is fed into the neural network, where the ICA algorithm performs the source separation and individual signals are obtained from the set of mixed signals.

Results

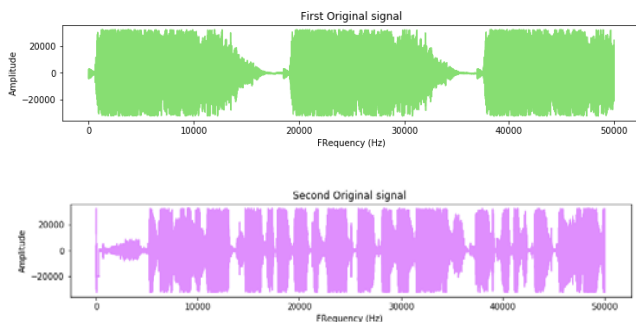


Figure 6(a): Plots of two original signals

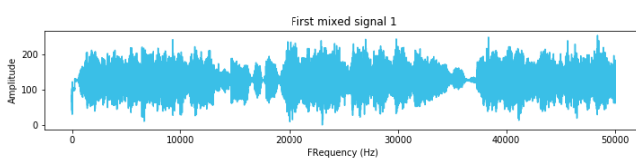


Figure 6(b): Plot of mixed signals

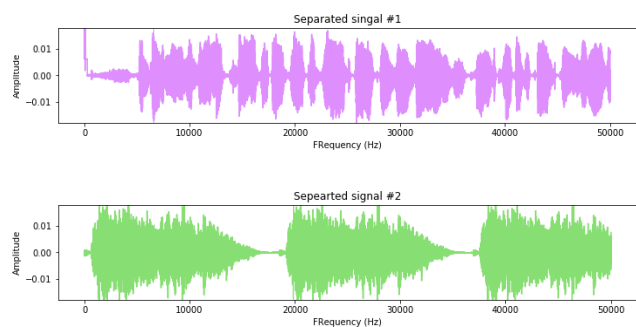


Figure 7: Plots of output separated signals

Conclusion

Inevitability is the essential property for a system used for recovering hidden data, noise cancellation and source signal separation from a set of mixed signals. In this paper, we have explained one of the supervised techniques of source separation in terms of invariability of the system and how the basic property of ICA depends on the Inevitability of the system. To support the concept of invariability of an ICA algorithm an experimental work has been carried out, where a neural network has been designed to separate the source signals from a two component mixed signal. The designed neural network is capable of separating the individual signals with less noise without losing any information. Thus, with this basic idea of ICA algorithm the implementation of BSS becomes very convenient and easy. We can claim that the composite system of Blind source separation using ICA algorithm is an invertible system and provides optimal results in Blind Source Separation.

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